On the Higher Order Modes of Elliptical Optical Fibers

SAAD MICHAEL SAAD, SENIOR MEMBER, IEEE

Abstract —**The point-matching numerical method is here employed for the modal analysis of the elliptical optical fiber of any eccentricity. Good agreement with other analytical and numerical methods is obtained. Previous disagreement in the literature is resolved.**

I. Introduction

SINGLE-MODE ELLIPTICAL fibers are recognized as an attractive option for optical communication and instrumentation. To assure single-mode operation, determination of the first higher order mode is of special importance.

Few analytical methods were developed to obtain the cutoff frequencies of higher order modes in the isotropic homogeneous elliptical fiber of Fig. 1. Lyubimov *et al.* [1], Yeh [2], Cozens and Dyott [3], and Rengarajan and Lewis [4], [5] solved the characteristic equation in terms of Mathieu functions. Also, most of the numerical methods developed for the arbitrarily-shaped fiber [6] can be readily applied to the elliptical case.

While there seems to be a serious disagreement among the analytical methods as explained in [4], [5], [7], the emphasis in the numerical methods was expectedly on efficiency and accuracy of the method rather than the degree of ellipticity of the fiber or its higher order mode content.

This paper, in contrast, concerns itself primarily with the cutoff phenomenon of higher order modes in the elliptical fiber in its full range of ellipticity ($0 \leqslant b/a \leqslant 1$). This is performed by employing the point-matching method. The paper also attempts to resolve the above-mentioned disagreement among analytical methods.

II. The Problem

There seems to be two procedures commonly followed to compute the cutoff frequency of higher order modes. Descriptively, one may call them the limit procedure and the direct procedure.

The limit procedure involves solving the fiber characteristic equation and numerically tracing the $\beta - V$ curve to the cutoff point ($\beta = k_0 n_2$, $V = V_c$), where

$$V = b\left(k_1^2 + k_2^2\right)^{1/2} \qquad (1)$$

$$k_1 = \left(k_0^2 n_1^2 - \beta^2\right)^{1/2} \qquad (2a)$$

$$k_2 = \left(\beta^2 - k_0^2 n_2^2\right)^{1/2} \qquad (2b)$$

Manuscript received December 18, 1984; revised June 14, 1985.
The author is with the Andrew Corporation, Orland Park, IL 60462.

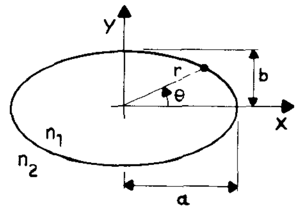

Fig. 1. Two-layer elliptical fiber.

β is the longitudinal, and k_1 and k_2 are the radial propagation constants. n_1 and n_2 are the refractive indexes of the core and cladding, respectively. This procedure has two advantages; it yields all possible higher order modes, and the solution can be examined throughout its applicable range of V not only at V_c. A serious limitation, however, is that the solution of the characteristic equation invariably involves the computation of a truncated infinite determinant that becomes increasingly ill-conditioned as the cutoff point is approached. This procedure is reported in the majority of references that deal with the fiber modal analysis, including the analytical work in [2], [4], [5] and the numerical work of this paper and of Eyges *et al.* [8] which is utilized, together with other references, to verify our curves.

The direct procedure, in contrast, seeks to reduce the characteristic equation to a simpler form representing the cutoff condition, thus improving the numerical condition of the determinant. In doing so, Lyubimov *et al.* [1] mathematically proved the existence of a mode type (what they called B-branch) for which V_c is determined by the roots of the Mathieu functions Ce and Se. On the numerical techniques side, Chiang [9] developed a finite-element method for the direct determination of V_c.

In an alternative direct procedure, Cozens and Dyott [3] reduced Yeh's exact characteristic equation [10] into the cutoff condition $\mathrm{Ce}(V) = 0$ and provided a curve shown in Fig. 2. That curve, however, was questionable to Citerne [7] and Rengarajan and Lewis [4], [5], for the cutoff condition was derived in [3] using the assumption that the elliptical fiber can support TE and TM modes, an approximation which is valid only in the near-circular case. Using the limit procedure as applied to the exact characteristic equation, [4] provided a new cutoff frequency curve, also shown in Fig. 2, which is lower than that of $\mathrm{Ce}(V) = 0$, thus limiting the fiber to a narrower bandwidth.

From [4], [5], [7]–[9], it is evidenced that the cutoff condition $\mathrm{Ce}(V) = 0$ given by [1] and [3] is indeed a valid

0018-9480/85/1100-1110$01.00 ©1985 IEEE

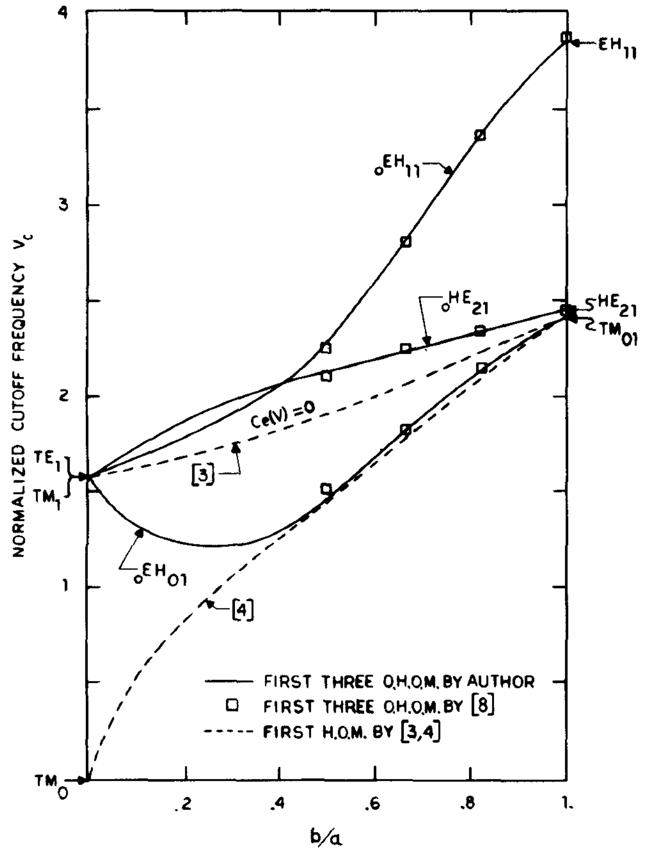

Fig. 2. Normalized cutoff frequency for odd higher order modes (OHOM) versus b/a in an elliptical fiber. $n_1 = 1.46$, $n_2 = 1.34$.

approximation for small eccentricity. On the other hand, the curve by [4] seems to be consistent with theoretical [8], [9] and experimental [11] data in the range $0.45 \leqslant b/a \leqslant 1.0$. For smaller b/a, however, there seems to be no confirmation of either curve by experiment or any other theoretical method.

III. Point-Matching Solution

In order to resolve independently such a paradox, this author adapted Goell's point-matching approach [12] to the present elliptical cross section of Fig. 1. Such an exercise was reported before for the same two-layer elliptical fiber [13], and even for the composite structure with two elliptic boundaries [14], though in both cases only small eccentricities were treated. In the point-matching method, the fields are first represented by an infinite expansion of circular harmonics, i.e.,

$$E_{z1} = \sum_{n=0}^{\infty} a_n \sin(n\theta + u_e) J_n(k_1 r) \exp(j\beta z) \quad \text{(3a)}$$

$$H_{z1} = \sum_{n=0}^{\infty} b_n \sin(n\theta + u_h) J_n(k_1 r) \exp(j\beta z) \quad \text{(3b)}$$

inside the fiber, and

$$E_{z2} = \sum_{n=0}^{\infty} c_n \sin(n\theta + u_e) K_n(k_2 r) \exp(j\beta z) \quad \text{(3c)}$$

$$H_{z2} = \sum_{n=0}^{\infty} d_n \sin(n\theta + u_h) K_n(k_2 r) \exp(j\beta z) \quad \text{(3d)}$$

in the cladding. Then the point-matching technique is applied around the boundary to generate a characteristic equation in β, which is to be solved for possible eigenvalues. The details of the method are quite intricate, but since they are described in detail for the rectangular guide case [12] and briefly for other cross sections [13]–[16], no further elaboration will be given here. Our computer programs also include computation of all components of electric and magnetic fields and plots of transverse fields. This is necessary for the classification of HE versus EH modes and also for checking on the solution lest it does not correspond to a physical solution, or misses a mode due to unfavorable numerical conditions. As an initial check on the accuracy of our programs, they were executed for several of the many special cases reported in the literature, e.g., [1]–[5], [8], [9], [13]. Our $\beta - V$ curves were in good agreement with the literature except in the cases where there has been already serious disagreement in such literature.

By executing our programs for the case reported in [4], namely, $n_1 = 1.46$ and $n_2 = 1.34$, we obtained the three solid curves shown in Fig. 2. These curves belong to the odd modes, i.e., those permitting a magnetic wall in the $x-z$ plane. The curves that belong to the even modes (not shown), i.e., those permitting an electric wall in the $x-z$ plane, are very close to their corresponding curves of the odd modes, and hardly distinguishable from them near the limits $b/a = 0$ and 1. In all computations, the convergence of the solution was carefully examined as N, the number of matched points in one quadrant of the ellipse, was increased from 12 to 36. Optimum N was found to vary with b/a.

Our cutoff values shown in Fig. 2, and also our field plots (not shown), are consistent with the expectation that the first and third higher order modes of that symmetry group, namely the $_o\mathrm{EH}_{01}$ and $_o\mathrm{EH}_{11}$, transform into the TM_{01} and EH_{11} modes, respectively, of the circular rod when $b/a = 1$, and into the TM_1 mode of the infinite symmetrical slab when $b/a = 0$. The second higher order mode of that group, namely the $_o\mathrm{HE}_{21}$, transforms into the HE_{21} mode of the circular rod, and into the TE_1 mode of the infinite slab.

The fact that our curves of Fig. 2 for the $_o\mathrm{EH}_{01}$, $_o\mathrm{HE}_{21}$ and $_o\mathrm{EH}_{11}$ modes converge toward the slab exact solution, namely, $V_c = \pi/2$, may be considered as a proof of the accuracy of our approach in the range of small b/a. As to the range $0.45 \leqslant b/a \leqslant 1.0$, our $_o\mathrm{EH}_{01}$ curve is in good agreement with data given in [4], [8], [9], and our $_o\mathrm{HE}_{21}$ and $_o\mathrm{EH}_{11}$ curves are in good agreement with data given in [8], [9].

IV. Resolving the Disagreement Between Different Methods

The disagreement between our $_o\mathrm{EH}_{01}$ curve and that of [4] in the range $0 \leqslant b/a \leqslant 0.4$ could be attributed possibly to inaccuracy in one or both methods. Based on the above-mentioned verification, however, the error in our method may expectedly be of small magnitude. On the other hand, the Mathieu function expansion of [4] and [5]

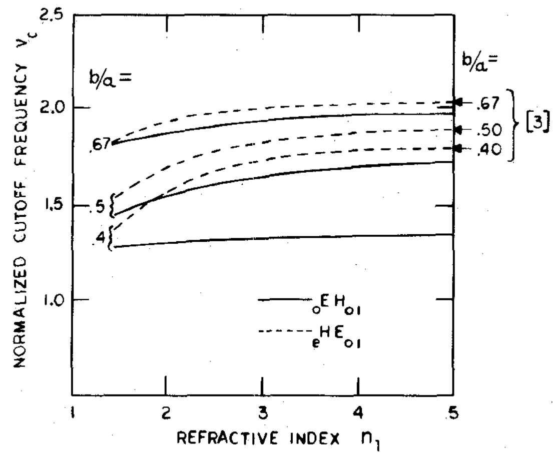

Fig. 3. Normalized cutoff frequency of the $_oEH_{01}$ and $_eHE_{01}$ modes versus n_1 for an elliptical fiber. $n_2 = 1.34$.

seems to suffer larger truncation error as b/a decreases. In fact, such solution, and many other numerical and analytical ones, suffer from a common source of error, namely, the characteristic equation has a singular determinant with some infinite elements at cutoff. The accuracy of the curve in [4] also may be questioned according to the fact that the $_oEH_{01}$ mode cannot converge toward the slab TM_0 mode because each is characterized by different symmetry planes.

Another possible reason for the disagreement between this paper and [3] on one side, and [4] and [5] on the other, may be explained by their respective consideration of the manner by which b and a vary. Obviously, the curves of Fig. 2 can be obtained by varying a and holding b constant, or varying b and holding a constant. Each consideration results in a particular physical structure. In the limit, where $b/a = 0$, our case, and also that of [3], is an infinite slab with finite thickness $2b$ supporting a spectrum of TE and TM modes with $V_c = n\pi/2$. The case of [4] and [5], in contrast, may be that of a slab with zero thickness and finite width $2a$, i.e., a slab that vanishes to allow the propagation of plane TEM waves with $V_c = 0$.

As to the cutoff curve $\mathrm{Ce}(V) = 0$ given by [3], it can now be claimed as an approximate solution which is valid not only in the quasi-circular case as suggested in [4] and [7], but also in the quasi-planar case. It is exact at the two limits, namely, the circular rod and the infinite slab. By varying $(n_1 - n_2)$ and tracing our point-matching solution for different b/a ratios, as shown in Fig. 3, it becomes evident that the solution in [3], which itself is independent of n_1 and n_2, represents an asymptotic solution for the $_eHE_{01}$ (rather than the $_oEH_{01}$) mode in an elliptical dielectric waveguide of any eccentricity. Such an asymptote is approached as $(n_1 - n_2)$ increases, a case encountered in surface-waveguide structures, rather than a weakly-guiding optical fiber.

V. Conclusion

The accuracy of a point-matching approach has been proven sufficient for the modal analysis of the elliptical optical fiber of any eccentricity. By computing the cutoff frequency of several higher order modes using this independent method, a serious dispute between other methods in the literature has been resolved. Our curves thus represent a successful attempt toward a better understanding of the cutoff characteristics of the elliptical fiber.

References

[1] L. A. Lyubimov, G. I. Veselov, and N. A. Bei, "Dielectric waveguide with elliptical cross section," *Radio Eng. Electron.* (USSR), vol. 6, pp. 1668–1677, 1961.
[2] C. Yeh, "Modes in weakly guiding elliptical optical fibers," *Opt. Quantum Electron.*, vol. 8, pp. 43–47, 1976.
[3] J. R. Cozens and R. B. Dyott, "Higher-mode cutoff in elliptical dielectric waveguides," *Electron. Lett.*, vol. 15, no. 18, pp. 558–559, Aug. 1979.
[4] S. R. Rengarajan and J. E. Lewis, "First higher-mode cutoff in two-layer elliptical fibre waveguides," *Electron. Lett.*, vol. 16, no. 7, pp. 263–264, Mar. 1980.
[5] ——, "Single mode propagation in multi-layer elliptical fiber waveguides," *Radio Sci.*, vol. 16, no. 4, pp. 541–547, July–Aug. 1981.
[6] S. M. Saad, "Review of numerical methods for the analysis of arbitrarily-shaped microwave and optical dielectric waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 894–899, Oct. 1985.
[7] J. Citerne, J. R. Cozens, and R. B. Dyott, "Comment on higher-mode cutoff in elliptical dielectric waveguides," *Electron. Lett.*, vol. 16, no. 1, pp. 13–14, Jan. 1980.
[8] L. Eyges, P. Gianino, and P. Wintersteiner, "Modes of dielectric waveguides of arbitrary cross sectional shape," *J. Opt. Soc. Am.*, vol. 69, no. 9, pp. 1226–1235, Sept. 1979.
[9] K. S. Chiang, "Finite element method for cutoff frequencies of weakly guiding fibres of arbitrary cross-section," *Opt. Quantum Electron.*, vol. 16, no. 6, pp. 487–493, Nov. 1984.
[10] C. Yeh, "Elliptical dielectric waveguides," *J. Appl. Phys.*, vol. 33, no. 11, pp. 3235–3243, Nov. 1962.
[11] S. C. Rashleigh and M. J. Marrone, "Polarization holding in elliptical-core birefringent fibers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1503–1511, Oct. 1982.
[12] J. E. Goell, "A circular-harmonic computer analysis of rectangular dielectric waveguides," *Bell Syst. Tech. J.*, vol. 48, pp. 2133–2160, Sept. 1969.
[13] E. Yamashita, K. Atsuki, O. Hasimoto, and K. Kamijo, "Modal analysis of homogeneous optical fibers with deformed boundaries," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 352–356, Apr. 1979.
[14] E. Yamashita, K. Atsuki, and Y. Nishino, "Composite dielectric waveguides with two elliptic-cylinder boundaries," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 987–990, Sept. 1981.
[15] J. R. James and I. N. L. Gallett, "Point-matched solutions for propagating modes on arbitrarily-shaped dielectric rods," *Radio Electron. Eng.*, vol. 42, no. 3, pp. 103–113, Mar. 1972.
[16] E. Yamashita, K. Atsuki, and R. Kuzuya, "Composite dielectric waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 986–990, Sept. 1980.

Saad Michael Saad (M'77–SM'82) was born in Alexandria, Egypt, on February 11, 1945. He received the B.Sc. and M.Sc. degrees from Alexandria University, Egypt, in 1965 and 1969, respectively, and the Ph.D. degree from the University of London, England, in 1974, all in electrical engineering.

From 1965 to 1977, he was with the Electronics Research Institute of the National Research Center, Cairo, Egypt, except for the period 1970–1973, when he was with the Department of

Electronic and Electrical Engineering of University College, London, England. During 1977–1978, he was a Research Associate at the Remote Sensing Laboratory of the University of Kansas. Since 1978, he has been with the Andrew Corporation, Orland Park, IL, where he is now Section Leader—Waveguide Components.

Dr. Saad's professional career has been primarily devoted to research and development in microwave passive components and antennas, with emphasis on numerical techniques and computer-aided design. He also worked as an Adjunct Assistant Professor at Ain Shams University (1975–1976) and the Military Technical College (1974–1976), both in Cairo, Egypt. He is now an Adjunct Associate Professor at the University of Illinois at Chicago.

Dr. Saad served as Secretary-Treasurer, Vice Chairman, and Chairman of the joint Chicago Chapter of the IEEE Antennas and Propagation Society and the Microwave Theory and Techniques Society (1981–1984). He is a Professional Engineer registered in the State of Illinois, holds several patents, and has authored and coauthored many technical publications.